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A game-theoretic analysis of the political structure of the Netherlands Antilles

Imma Curiel

Dept. of Math. and Stat., UMBC, 1000 Hilltop Circle, Baltimore, MD 21250, USA
(e-mail: curiel@math.umbc.edu; romali@attglobal.net)

Abstract. This paper uses apportionment methods and game theory to analyze the political structure of the Netherlands Antilles both before and after Aruba's Status Aparte. A comparison is made among the islands of the position of a voter on each island with respect to representation and voting power.

Key words: Apportionment method, Fair representation, Power index

1 Introduction

The Netherlands Antilles are five Dutch islands in the Caribbean Sea. They are Curaçao, Bonaire, Sint Maarten, Sint Eustatius, and Saba. Together with Aruba they form the Dutch West-Indies. Prior to 1986 Aruba was also part of the Netherlands Antilles. On January 1, 1986 Aruba obtained the so called "Status Aparte" and became a separate entity in the Dutch Kingdom. In this paper the term "The Netherlands Antilles" will mean the six islands if we are speaking about the time before Aruba's Status Aparte, and the five islands otherwise.

Until 1936 the Netherlands Antilles were a colony with all governmental power vested in a governor who was appointed by the Dutch government and without any elected representation of the population in the government. In 1936 partial suffrage was instituted. In 1948 universal suffrage was introduced and the first general elections were held on March 17, 1949. These islands with their small but diverse population, separate geographical locations (the distance between the windward islands Sint Maarten, Sint Eustatius, and Saba, and the leeward islands, Curaçao, Aruba, and Bonaire is approximately 800 kilometers), and different historical backgrounds provide us with an interesting scenario for a game theoretical analysis of their political structure.

This is not solely an analysis of the past. The five remaining islands of the Netherlands Antilles are holding a continuing debate with each other and the Netherlands to arrive at a political structure that is less cumbersome and expensive than the current one, and that is perceived by all parties concerned to be more fair.

In this paper we will look at four well-known methods of fair representation and compare the apportionments that they prescribe with the actual one used in the parliament of the Netherlands Antilles. We will also analyze the situation using simple games and power indices. In section 2 we will consider apportionment methods for the period 1948–1982 when Aruba was still in the Netherlands Antilles. Section 3 looks at the same period using simple games and power indices. In section 4 we consider apportionment methods and simple games and power indices for the period after 1985 when Aruba had left the Netherlands Antilles. Section 5 ends the paper with some concluding remarks and acknowledgments.

The data used in sections 2, 3 and 4 are from Reinders [8].

2 Four apportionment methods for the period 1948–1982

In 1948 the Netherlands Antilles obtained for the first time some form of autonomy with the institution of an elected parliament and a governmental body that was accountable to the parliament. The distribution of seats in that first parliament was: Curaçao 8, Aruba 8, Bonaire 2, Sint Maarten 1, Sint Eustatius 1, and Saba 1. Voters on island X could and can only vote for political parties of island X. Striking is the fact that Curaçao and Aruba were assigned the same number of seats in spite of the large difference between their population sizes. Unfortunately, Curaçao is the only island for which the number of people who were eligible to vote is known for the election of March 17, 1949 but by using the numbers of the election of December 21, 1950 we can still obtain some idea of the discrepancy in the number of people represented by the members of parliament from the various islands. The number of people eligible to vote in the election of 1950 were for Curaçao: 39,768, for Aruba: 14,250, for Bonaire: 2,219, and for the three Windward Islands together: 1,398. This means that each MP (= Member of Parliament) from Curaçao represented approximately 4,971 votes, each MP from Aruba 1,781 votes, each MP from Bonaire 1,110 votes, and each MP from the Windward Islands 466 votes. These numbers are summarized in Table 1. This first distribution of seats illustrates the following difficulty in the formation of a parliament for the six islands. Because of the extremely small population size of the Windward Islands compared to that of Curaçao strict proportional representation will give rise

Table 1. Approximate nr. of voters represented by an MP in 1949

	Curaçao	Aruba	Bonaire	WI
nr. of eligible voters	39,768	14,250	2,219	1,398
nr. of seats in parliament	8	8	2	3
nr. of votes repr. by each MP	4,971	1,781	1,110	466

to a parliament size that is ridiculously large for the number of people that has to be represented.

This distribution of seats didn't survive long. In 1950 it was changed to 12 seats for Curaçao, 8 for Aruba, 1 for Bonaire, and 1 for the three Windward Islands together. It is not clear what considerations led to exactly this distribution. In the following we will discuss four well-known apportionment methods and we will compute the distribution of seats prescribed by these methods for the period 1948–1982 (1982 being the last time that a parliament was elected for the six islands.)

The apportionment problem, that is, how to distribute an integer number of resources among several participants in integer amounts arises in several situations and has received ample attention in the literature. Cf. Balinski and Young [1], Lucas [6]. The fair representation problem is an example of an apportionment problem that arises when seats in a parliament have to be distributed among electorial units, in this case the islands, proportionally to the sizes of the respective electorates.

It has been established that it is impossible for any apportionment rule to satisfy all desirable properties that one would like it to have. Still, one cannot get around using a rule in practice and here we will look at four rules that have received quite some attention in practice as well as in theory. They are known under more than one name but the names that we will use for them in this paper are:

1. The Method of Greatest Remainders (GR)
2. The Method of Smallest Critical Multiplier (SCM)
3. The Method of Arithmetic Mean (AM) and
4. The Method of Geometric Mean (GM)

In the description of these methods we will use the following notation. Let $N = \{1, 2, \ldots, n\}$ be the set of participants. The number of seats that has to be distributed is denoted by P. The number of eligible voters of each participant i is denoted by p_i and the quota q_i of each participant i is given by

$$q_i = \frac{p_i P}{\sum_{i \in N} p_i}$$

For every $x \in R$ the greatest integer less than or equal to x will be denoted by $\lfloor x \rfloor$ and the smallest integer greater than or equal to x will be denoted by $\lceil x \rceil$.

The method of greatest remainders

The Method of Greatest Remainders starts with assigning $\lfloor q_i \rfloor$ to i. In general, there will be seats left over. These are assigned according to greatest remainders. That is, the participant i with the greatest remainder, $q_i - \lfloor q_i \rfloor$, gets a seat. If there are more seats left then the participant with the next greatest remainder gets a seat and so on.

With this method the distribution of seats in 1948 would have been Curaçao: 14, Aruba: 5, Bonaire: 1, and WI: 1.

The method of smallest critical multiplier

The Method of Smallest Critical Multiplier also starts with assigning $\lfloor q_i \rfloor$ to i. Then the multiplier

$$m_i = \frac{\lfloor q_i \rfloor + 1}{q_i}$$

is computed for each $i \in N$ and M is taken to be equal to the smallest m_i. Each q_i is adjusted to $\hat{q}_i = M \times q_i$ and i is assigned $\lfloor \hat{q}_i \rfloor$ seats. If all seats are still not assigned the process is repeated with the new quotas.

With this method the distribution of seats in 1948 would have been Curaçao: 16, Aruba: 5, Bonaire: 0, and WI: 0.

The method of arithmetic mean

Whereas the two previous methods both start with rounding down, the Method of Arithmetic Mean rounds up or down depending on the difference between q_i and the arithmetic mean of $\lfloor q_i \rfloor$ and $\lceil q_i \rceil$. The number of seats assigned to i is

$$\begin{cases} \lfloor q_i \rfloor & \text{if } \ q_i < \frac{\lfloor q_i \rfloor + \lceil q_i \rceil}{2} \\ \lceil q_i \rceil & \text{otherwise} \end{cases}$$

If this assignment results in too many seats being distributed then

$$m_i = \frac{n_i - 0.5}{q_i}$$

is computed for every $i \in N$ and the participant i with m_i closest to 1 loses 1 seat. Here n_i is the number of seats assigned to i. If necessary, this is repeated with the new assignment of seats.

If the original assignment results in too few seats being distributed then

$$m_i = \frac{n_i + 0.5}{q_i}$$

is computed for every $i \in N$ and the participant i with m_i closest to 1 gets an extra seat. Of course, this is also repeated if necessary.

With this method the distribution of seats in 1948 would have been Curaçao: 14, Aruba: 5, Bonaire: 1, and WI: 1.

The method of geometric mean

The Method of Geometric Mean also rounds up or down but depending on the difference between q_i and the geometric mean of $\lfloor q_i \rfloor$ and $\lceil q_i \rceil$. The number of seats assigned to i is

$$\begin{cases} \lfloor q_i \rfloor & \text{if } \ q_i < \sqrt{\lfloor q_i \rfloor \lceil q_i \rceil} \\ \lceil q_i \rceil & \text{otherwise} \end{cases}$$

If this results in too many seats being assigned then

$$m_i = \frac{\sqrt{n_i(n_i - 1)}}{q_i}$$

is computed for every $i \in N$ and the participant i with m_i closest to 1 loses 1 seat. The process is repeated if necessary.

If too few seats are assigned then

$$m_i = \frac{\sqrt{n_i(n_i + 1)}}{q_i}$$

is computed for every $i \in N$ and the participant i with m_i closest to 1 receives 1 extra seat. This is repeated if necessary. Note that this method guarantees at least 1 seat to every participant.

With this method the distribution of seats in 1948 would have been Curaçao: 14, Aruba: 5, Bonaire: 1, WI: 1.

In Table 2 the assignment of seats according to the four apportionment methods discussed above is given, together with the number of votes represented by an MP of each island under the actual seat distribution of Curaçao: 12, Aruba: 8, Bonaire: 1, and WI: 1, for the period 1950–1982. The number of eligible voters on the Windward Islands in 1979 is not known. Studying the table we see that Curaçao has received at least 2 seats too few according to each of the four apportionment methods during the period 1950–1982. Aruba, on the other hand has received at least 2 seats too many during the same period.

As mentioned above the GM method guarantees at least one representative to each island and therefore it is an appropriate method for distributing seats among the islands. Another nice property of the GM method is that it minimizes the relative difference in number of votes represented by an MP, for any two islands.

The table illustrates the fact that the distribution of seats should be updated periodically to account for changes in the sizes of the electorates.

3 Simple games and power indices

In section 2 we analyzed the problem of allocating seats in the parliament of the Netherlands Antilles among the six islands from the point of view of fair representation using apportionment rules. In this section we will look at this problem using simple games and power indices. Simple games have been used to evaluate the distribution of power in voting situations and situations involving committee control since their introduction by Shapley and Shubik [9]. A cooperative game $< N, v >$ is called a simple game if $v(N) = 1$ and $v(S) \in \{0, 1\}$ for all $S \in 2^N$. A coalition S in a simple game is called winning if $v(S) = 1$ and losing if $v(S) = 0$. A weighted majority game is a simple game which is fully described by its quota q and weights w_i for all $i \in N$. A coalition S in a weighted majority game is winning if $\sum_{i \in S} w_i \geq q$, otherwise S is losing. A weighted majority game is denoted by $[q; w_1, w_2 \ldots, w_n]$.

Several power indices have been proposed in the literature as appropriate measures of power in situations modeled by simple games. Owen [7] uses simple games and power indices to analyze the method for electing a president

Table 2. Distribution of seats according to 4 apportionment methods during 1950–1982

	Curaçao	Aruba	Bonaire	Windward Islands
1950	3314	1781	2219	1398
GR	15	5	1	1
SCM	16	6	0	0
AM	15	5	1	1
GM	15	5	1	1
1954	3697	2043	2490	1250
GR	15	6	1	0
SCM	16	6	0	0
AM	15	6	1	0
GM	15	5	1	1
1958	4136	2330	2623	1525
GR	15	6	1	0
SCM	16	6	0	0
AM	15	6	1	0
GM	15	5	1	1
1962	4247	2572	2706	1640
GR	15	6	1	0
SCM	16	6	0	0
AM	15	6	1	0
GM	14	6	1	1
1966	4552	2836	3076	2092
GR	15	6	1	0
SCM	16	6	0	0
AM	14	6	1	1
GM	14	6	1	1
1969	5315	3257	3564	2511
GR	15	6	1	0
SCM	16	6	0	0
AM	14	6	1	1
GM	14	6	1	1
1973	6648	4117	4379	3864
GR	14	6	1	1
SCM	16	6	0	0
AM	14	6	1	1
GM	14	6	1	1
1977	7473	4619	4966	5644
GR	14	6	1	1
SCM	15	6	0	1
AM	14	6	1	1
GM	14	6	1	1
1979	7945	4865	5202	–
1982	8515	5263	5636	7504
GR	14	6	1	1
SCM	15	6	0	1
AM	14	6	1	1
GM	14	6	1	1

in the United States. In this paper we will restrict our attention to the Banzhaf index introduced in Banzhaf [2].

To pass a bill in the parliament of the Netherlands Antilles a simple majority of votes is needed. If we assume that all the MPs from an island vote “en bloc” we can model this situation as a weighted majority game and we can obtain some idea of the political power of each island by computing the Banzhaf power index for this game.

The simple game associated with the situation in 1948 is the weighted majority game [11;8,8,2,1,1,1]. The first number is the quota, that is, the number of votes needed to pas a bill. The other six numbers represent the weights of the islands in the order Curaçao, Aruba, Bonaire, Sint Maarten, Sint Eustatius, Saba.

The Banzhaf power index counts the number of times that a player's vote is crucial. A player's vote is said to be crucial if by changing it he changes the outcome of the game. It considers all 2^n possible combinations of "yes" and "no" votes and it counts the number of combinations in which a player is a swing, that is, in which he can change the outcome by changing his vote. Let b_i be the number of combinations in which player i is a swing. Formally,

$$b_i = \sum_{S \ni i} (v(S) - v(S \setminus \{i\})) + \sum_{S, N \setminus S \ni i} (v(S \cup \{i\}) - v(S))$$

The normalized Banzhaf power index β is given by

$$\beta_i = \frac{b_i}{\sum_{i=1}^{n} b_i}$$

Another normalization studied by Dubey and Shapley [3] divides by 2^n rather than by $\sum_{i=1}^{n} b_i$ yielding the index

$$\beta'_i = \frac{b_i}{2^n}$$

This index is called the Banzhaf measure of voting power by Felsenthal and Machover [4]. The normalized Banzhaf power index for the parliament of 1948 is $(\frac{2}{7}, \frac{2}{7}, \frac{3}{14}, \frac{1}{14}, \frac{1}{14}, \frac{1}{14})$. So the first parliament of the Netherlands Antilles gave the same power to the eligible voters of Curaçao as to those of Aruba although the former group was approximately three times as large as the latter group. Bonaire with an electorate that was approximately fifteen times smaller than that of Curaçao had a power that was three-quarters of that of Curaçao. The Windward Islands taken together had an electorate that was almost half of that of Bonaire but the same power as Bonaire.

We have already seen that this first parliament was not designed by taking into account considerations of fair representation. In hindsight (the theory of simple games and power indices was not around in 1948) we see that it didn't distribute power fairly either.

What can we say about the subsequent years? The weighted majority game that describes the situation during the period 1950–1982 is [12;12,8,1,1]. The last four numbers represent the weights of the islands in the order Curaçao, Aruba, Bonaire, Windward Islands. One sees immediately that Curaçao is a dictator. If all its MPs vote the same way it doesn't need anybody else to pass a bill. The Banzhaf power index is (1,0,0,0) assigning all power to Curaçao. The situation has moved to the other extreme it seems. From giving too much power to the smaller islands to giving all the power to Curaçao. Recall that even the 12 MPs of Curaçao did not constitute a fair representation. The number should have been at least 14 during the period under consideration. See Table 2 in section 2. So taking care of the fair representation issue would only make matters worse with respect to the power issue.

In the following we will try to design a parliament in a way that takes care of both these aspects. Our first goal is to come up with a parliament for which the power of each voter on each island is the same when measured by the Banzhaf power index. We will do this for the year 1950 since that was the year in which the composition of the parliament of the Netherlands Antilles, that stayed in place till the Status Aparte of Aruba, was established.

From Felsentahl and Machover [4] (theorem 3.4.3, page 66) it follows that the power of each voter on each island as measured by β' is equal if the vector $(\beta'_C, \beta'_A, \beta'_B, \beta'_{WI})$ is proportional to the vector $(\sqrt{n_C}, \sqrt{n_A}, \sqrt{n_B}, \sqrt{n_{WI}})$. Here n_i is the size of the electorate of island i. Since

$$\beta = \frac{2^n}{\sum_{i=1}^{n} b_i}\beta'$$

it follows that the above also holds when the power index β is used. In 1950 the Windward Islands had 1,398 eligible voters. By looking at the ratios of the square root of this number and the square root of the size of the group of eligible voters of the other islands we obtain an idea of what the Banzhaf power index should ideally be to satisfy our goal. These ratios are given below.

$$\sqrt{\frac{C}{WI}} = \sqrt{\frac{39,768}{1,398}} = 5.33$$

$$\sqrt{\frac{A}{WI}} = \sqrt{\frac{14,250}{1,398}} = 3.19$$

$$\sqrt{\frac{B}{WI}} = \sqrt{\frac{2,219}{1,398}} = 1.26$$

It is clear that we cannot construct a simple game which has a Banzhaf power index that is exactly proportional to $(5.33, 3.19, 1.26, 1)$. The best we can do is the simple game with minimal winning coalitions {A,C} and {B,C,WI}. This makes a veto player but not a dictator of Curaçao. The Banzhaf power index of this game is $\frac{1}{10}(5, 3, 1, 1)$. There are several weighted majority games that will have these minimal winning coalitions. If we want to keep the number of seats in the parliament equal to 22 the game [17;15,5,1,1] would be appropriate. So Curaçao is assigned 15 seats, Aruba 5, Bonaire 1, and the Windward Islands 1, but a simple majority is not enough to pass a bill. Seventeen of the twenty-two votes are needed for that. Note that three of the apportionment methods that we discussed in section 2 resulted in this distribution of seats among the islands for the year 1950.

We can approximate the relative influence of a voter on island i by using the following argument. In a population of n voters the number of times that a particular voter is a swing is

$$2\binom{n-1}{\frac{n-1}{2}} = \frac{2(n-1)!}{(\frac{n-1}{2}!)^2} \quad \text{if } n \text{ is odd, } 2\binom{n-1}{\frac{n}{2}} = \frac{2(n-1)!}{\frac{n}{2}!\frac{n-2}{2}!} \quad \text{if } n \text{ is even.}$$

Using Stirling's formula

$$n! \approx \sqrt{2\pi n}n^n e^{-n}$$

Table 3. A comparison of the $\sqrt{\frac{2}{\pi n_i}}\beta_i$'s for the period 1950–1982

	C	A	B	WI
1950	$20 \cdot 10^{-4}$	$20 \cdot 10^{-4}$	$17 \cdot 10^{-4}$	$21 \cdot 10^{-4}$
1954	$19 \cdot 10^{-4}$	$19 \cdot 10^{-4}$	$16 \cdot 10^{-4}$	$23 \cdot 10^{-4}$
1958	$18 \cdot 10^{-4}$	$18 \cdot 10^{-4}$	$16 \cdot 10^{-4}$	$20 \cdot 10^{-4}$
1962	$18 \cdot 10^{-4}$	$17 \cdot 10^{-4}$	$15 \cdot 10^{-4}$	$20 \cdot 10^{-4}$
1966	$19 \cdot 10^{-4}$	$16 \cdot 10^{-4}$	$14 \cdot 10^{-4}$	$17 \cdot 10^{-4}$
1969	$16 \cdot 10^{-4}$	$15 \cdot 10^{-4}$	$13 \cdot 10^{-4}$	$16 \cdot 10^{-4}$
1973	$14 \cdot 10^{-4}$	$13 \cdot 10^{-4}$	$12 \cdot 10^{-4}$	$13 \cdot 10^{-4}$
1977	$13 \cdot 10^{-4}$	$12 \cdot 10^{-4}$	$11 \cdot 10^{-4}$	$11 \cdot 10^{-4}$
1979	–	–	–	–
1982	$12 \cdot 10^{-4}$	$12 \cdot 10^{-4}$	$11 \cdot 10^{-4}$	$9 \cdot 10^{-4}$

we obtain

$$\frac{2(n-1)!}{(\frac{n-1}{2}!)^2} \approx \sqrt{\frac{2}{\pi(n-1)}}2^n \quad \text{if } n \text{ is odd, and} \frac{2(n-1)!}{\frac{n}{2}!\frac{n-2}{2}!} \approx \sqrt{\frac{2}{\pi n}}2^n \quad \text{if } n \text{ is even.}$$

Dividing by 2^n gives us the approximate relative influence of each player, on his island, namely,

$$\sqrt{\frac{2}{\pi n}}$$

So the power of a voter on island i can be approximated by the above quantity times the power of the island, that is, $\sqrt{\frac{2}{\pi n_i}}\beta_i$.

In Table 3 these numbers are computed for the game described above for the period 1950–1982. We see that the $\sqrt{\frac{2}{\pi n_i}}\beta_i$'s do not differ much, indicating that this game distributes power equitably among the voters on the islands for the period 1950–1982.

We see that such an analysis as given in this section and the previous one would have enabled the construction of a parliament with a more equitable distribution of power among the islands. Of course, one would have had to deviate from the rule that a simple majority is enough to pass a bill.

The question arises why there was no strong outcry against the apparent dictatorship of Curaçao. Even without a theory of simple games and power indices it should have become clear to the other islands by observation that Curaçao was able to govern the Netherlands Antilles without needing their cooperation. To answer this question we have to look more closely at the political situations in the Netherlands Antilles during the period under consideration. What we see then is that for the period 1950–1966 there were two big political parties on Curaçao that never cooperated, effectively splitting the seats assigned to Curaçao in two. Cf. Reinders[8]. So instead of the game [12;12,8,1,1] the game [12;6,6,8,1,1] is what one observes when analyzing the political landscape during those years. This last game has Banzhaf power index equal to $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0, 0)$. This looks much better for Aruba which now has the same power as each of the two parties on Curaçao with a population of eligible voters that is less than half of that of Curaçao.

Table 4. Distr. of seats according to four apportionm. methods for 1985–1990

	C	B	M	E	S
1985	7837	2101	1913	834	648
GR	20	1	1	0	0
SCM	20	1	1	0	0
AM	20	1	1	0	0
GM	18	1	1	1	1
1990	8003	2178	2669	772	627
GR	20	1	1	0	0
SCM	20	1	1	0	0
AM	20	1	1	0	0
GM	18	1	1	1	1

The relationship between the two big political parties on Curaçao was so strained that, in fact, a simple game that excludes winning coalitions which contain both is a better description of the situation. If we denote the two parties of Curaçao by C1 and C2 the game [12;6,6,8,1,1] has minimal winning coalitions {C1,C2},{C1,A},{C2,A}. Now we look at the game with minimal winning coalitions {C1,A}, and {C2,A} only. The Banzhaf power index of this game is $(\frac{1}{6}, \frac{1}{6}, \frac{2}{3}, 0, 0)$. It assigns more power to Aruba than to each of the parties of Curaçao.

After 1966 the situation on Curaçao became diffuser with more political parties dividing the power and refusing to cooperate among each other. Although this closer look at what was happening explains why effectively, Curaçao did not have the power of a dictator it still leaves Bonaire and the Windward Islands without any power. Still when looking at what was really happening we see that every time at least one of them and most of the time both were participating in the government. This can be explained if we take into account that the perception was that on Aruba also the power had to be split between two parties although for most of the time one was really bigger than the other. The game that approximately describes this situation is [12;6,6,4,4,1,1] with Banzhaf power index $\frac{1}{14}(4, 4, 2, 2, 1, 1)$ So we see that political reality may have contributed to diffusing the power of Curaçao and Aruba somewhat. Still, it is not a good practice to count on disunity on an island to balance out power, when designing a distribution of seats in parliament among the islands.

4 After 1985

In 1986 Aruba obtained the Status Aparte that it had wanted from the moment that the Netherlands Antilles had become an autonomous region. From then on the Netherlands Antilles consisted of five islands. The election for parliament in 1985 was held among these five islands with Aruba holding a separate election for its parliament. Of course a new distribution of seats for the parliament of the Netherlands Antilles was necessary. It was decided to keep the number of seats equal to 22 and distribute them as follows. Curaçao: 14, Bonaire: 3, Sint Maarten: 3, Sint Eustatius: 1, Saba: 1.

Not surprisingly this distribution does not constitute either fair representation or fair distribution of power. In Table 4 we do the same for the

Table 5. $\sqrt{\frac{2}{\pi n_i}}\beta_i$'s for 1985 and 1990

	C	B	M	E	S
1985	$15 \cdot 10^{-4}$	$14 \cdot 10^{-4}$	$15 \cdot 10^{-4}$	$13 \cdot 10^{-4}$	$14 \cdot 10^{-4}$
1990	$15 \cdot 10^{-4}$	$14 \cdot 10^{-4}$	$13 \cdot 10^{-4}$	$14 \cdot 10^{-4}$	$15 \cdot 10^{-4}$

years 1985,1990 as we did in Table 2 for the period 1950–1982. That is, the numbers of voters represented by an MP is given for each island together with the distribution of seats according to the four apportionment methods discussed in section 2. Here C = Curaçao, B = Bonaire, M = Sint Maarten, E = Sint Eustatius, S = Saba. We see that the actual distribution is rather different from what these methods prescribe.

Analyzing the situation using simple games and power indices we see that Curaçao once again is a dictator. (If the political parties on the island could cooperate.) Similar to the way it was done in section 3 we will design and analyze a game that distributes power more equally. We will use the data of 1985. Below we compare the square roots of the sizes of the electorates on the five islands using Saba as a benchmark.

$$\sqrt{\frac{C}{S}} = \sqrt{\frac{109,713}{648}} = 13.01 \qquad \sqrt{\frac{B}{S}} = \sqrt{\frac{6,304}{648}} = 3.12$$
$$\sqrt{\frac{M}{S}} = \sqrt{\frac{5,738}{648}} = 2.98 \qquad \sqrt{\frac{E}{S}} = \sqrt{\frac{834}{648}} = 1.13$$

These numbers indicate that Curaçao should be a veto player, that Bonaire and Sint Maarten should have the same amount of power, and that Sint Eustatius and Saba should have the same amount of power. This leads to the simple game with minimal winning coalitions {C,B}, {C,M}, {C,E,S}. The Banzhaf power index for this game is $\frac{1}{21}(13, 3, 3, 1, 1)$. We see that the proportionality between this vector and the vector (13.01,3.12,2.98,1.13,1) is not bad at all. This leads us to believe that the $\sqrt{\frac{2}{\pi n_i}}\beta_i$'s will be almost equal. A weighted majority game with these minimal winning coalitions is, for example, [16;14,3,3,1,1].

In Table 5 we give the $\sqrt{\frac{2}{\pi n_i}}\beta_i$'s for 1985 and 1990. We see that the value $\sqrt{\frac{2}{\pi n_i}}\beta_i$ is almost the same for each island indicating that this game distributes voting power equitably.

5 Conclusion

In this paper we performed an analysis of the political structure of the Netherlands Antilles. We saw that the distributions of seats in parliament, both the ones that have been employed in the past as well as the one that is currently in place, do not constitute fair representation or fair distribution of power. We looked at four well known apportionment methods and saw that the GM method could serve for distributing the seats among the islands. As the electorate sizes of the islands change the distribution should be updated. Improving the situation with respect to the distribution of power involves demanding more than a simple majority of votes to pass a bill in Parliament.

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